1. $\sum_{n=4}^{\infty} \left(\frac{1}{3}\right)^n$ converges.
2. If $\sum a_n$ is absolutely convergent, then $\sum a_n$ is convergent.
3. $\int e^{2x}\, dx = \frac{C}{2} e^{2x}$.
4. If the nonhomogeneous part of a second-order, linear, constant-coefficient differential equation is $g(x) = e^x$, then $y_p(x) = Ae^x$ is always a valid form of the particular solution.
5. If $f(x) \leq g(x)$ and $\int_0^\infty g(x)\, dx$ diverges, then $\int_0^\infty f(x)\, dx$ diverges.
6. $\int \sin(3x)\cos(2x)\, dx = \frac{1}{2} \int \left[\sin(5x) + \sin(x)\right]\, dx$.
7. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$.
8. On the first day of the semester, Oski starts an ant farm. Let $A(t)$ be the number of ants in Oski's ant farm t days after the first day of the semester. If we model the ant population using the natural growth model with growth rate constant $k = 3$, then Oski's ant farm population adds 3 ants per day.
9. Since $-\frac{1}{n} \leq \frac{\sin(n)}{n} \leq \frac{1}{n}$ and $\lim_{n\to\infty} -\frac{1}{n} = \lim_{n\to\infty} \frac{1}{n} = 0$, it follows that $\lim_{n\to\infty} \frac{\sin(n)}{n} = 0$.
10. $\int_{-\infty}^{\infty} e^{-|x|}\, dx$ converges.
11. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ converges by the Ratio Test.
12. Euler's Method gives an exact solution to a differential equation.
13. $\int_1^\infty \frac{1}{x}\, dx$ converges.
14. $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}-2}$ converges by the Direct Comparison Test to $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$.
15. The logistic growth equation is a linear differential equation.
16. $\int_{-\pi/2}^{\pi/2} \csc(|x|)\, dx = \ln|\csc(\pi/2) - \cot(\pi/2)| - \ln|\csc(\pi/2) - \cot(\pi/2)| = 0$.
17. If we use $u = x^3$, then $\int_0^2 x^2 \cos(x^3)\, dx = \int_0^2 \frac{1}{3} \cos(u)\, du$.
18. If the interval of convergence of a power series is $[0, 4)$, then the radius of convergence is $R = 2$.
19. $y' = y - 2$ is a separable differential equation.
20. $$\int_0^{\pi/2} \sin^5(x)\cos^2(x)\, dx = \int_0^1 (1-u^2)^2 u^2\, du.$$
21. You can use integration by parts on improper integrals.
22. $\sum_{n=1}^{\infty} (-1)^n n^n$ diverges by the Root Test.
23. $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = 2$.
24. The third-order Taylor polynomial of $\sin x$ centered at $x = 0$ is $T_3(x) = x - \frac{x^3}{6}$.
25. Since $\lim_{n\to\infty} \frac{3\sqrt{n}+2+n^{-1}}{\sqrt{n}} = 3$, the series $\sum_{n=1}^{\infty} \frac{1}{3\sqrt{n}+2+n^{-1}}$ converges because $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges.
26. $\frac{x^2}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$ is a valid partial fraction decomposition.
27. The center of mass of the region between $y = x+1$, $y = 0$, $x = -1$, and $x = 1$ is in the first quadrant.
28. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\cos(\pi n) n^2}$ converges by the alternating series test.

29. In the method of variation of parameters, if the homogeneous solution is $y_h(x) = C_1e^{-x}\cos(3x) + C_2e^{-x}\sin(3x)$, then the particular solution is of the form $y_p(x) = u_1(x)e^{-x}\cos(3x)+u_2(x)e^{-x}\sin(3x)$.
30. A spring with a 3-kg mass and a damping constant 10 kg/s can be held stretched 0.5 meters beyond its natural length by a force of 2 newtons. The spring is then stretched 4 meters beyond its natural length and released with zero velocity. The spring constant is $k = 4/2 = 2$ newtons per meter.
31. $dG/dt = 3G(1 - 0.01G)$ describes logistic growth with carrying capacity 100.
32. $\int \frac{1}{x^2}\, dx = \ln(x^2) + C$.
33. If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ is divergent.
34. The Lotka-Volterra predator-prey equations always have exactly one equilibrium solution.
35. If the ratio test on $\sum_{n=0}^{\infty} a_n x^n$ says that the power series converges for $\left|x - \frac{1}{3}\right| < 1$, then the radius of convergence is $R = \frac{1}{3}$.
36. Every bounded, monotonic sequence is convergent.
37. $\sum_{n=0}^{\infty} \cos(n^2)\frac{3^n}{4^n}$ is absolutely convergent.
38. If $y^{1/2} = \sin^2(x) + C$, then $y = \sin^4(x) + C^2$.
39. A tank contains 1000 L of pure water. Brine that contains 0.05 kg of salt per liter of water enters the tank at a rate of 5 L/min. Brine that contains 0.04 kg of salt per liter of water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. If $S(t)$ is the kilograms of salt in the tank after t minutes, the initial-value problem that describes this situation is
$$\frac{dS}{dt} = 0.25 - 0.015S, \quad S(0) = 0.$$
40. $\sum_{n=0}^{\infty}(-3)^n = \frac{1}{1-(-3)} = \frac{1}{4}$.
41. The Maclaurin series of $f(x)$ is the Taylor series of $f(x)$ centered at $x = 0$.
42. $\frac{1}{3-x} = \frac{1}{1-(x/3)} = \sum_{n=0}^{\infty}(x/3)^n$.
43. $\sum_{n=1}^{\infty} n^{4/3}$ converges since $p = 4/3 > 1$.
44. $2 \cdot 4 \cdot 6 \cdots (2m) = 2^m m!$
45. Since $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$, the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges.
46. The auxiliary equation for $y'' + 2y' = 0$ is $r^2 + 2 = 0$.
47. If the Ratio Test is inconclusive, then you can still use a different test to determine if the series converges.
48. $\sum_{n=1}^{\infty}(-1)^n n$ diverges by the Root Test.
49. The radius of convergence of a power series can be zero.
50. If the recursion relation of a series solution $\sum_{n=0}^{\infty} c_n x^n$ to a second-order linear differential equation is $c_{n+2} = \frac{1}{2}c_n$, then the general solution is
$$y(x) = c_0 \sum_{m=0}^{\infty} \frac{1}{2^m} x^{2m} + c_1 \sum_{m=0}^{\infty} \frac{1}{2^m} x^{2m+1}.$$
51. If the homogeneous solution of a nonhomogeneous, second-order, linear, constant-coefficient differential equation is $y_h(x) = C_1e^{2x} + C_2e^{-x}$ and the nonhomogeneous part is $g(x) = x\cos x$, then a valid guess for a particular solution is $y_p(x) = (Ax + B)(C\cos x + D\sin x)$.

52. $\int \tan x \, dx = \sec^2 x + C$.

53. $y(x) = e^x - e^{-x+2}$ is a solution to $y'' - y = 0$.

54. Taylor's Inequality states that if $|f^{(n+1)}(x)| \leq M$ for $|x - a| < r$, then the remainder $R_n(x) = f(x) - T_n(x)$ of the Taylor series satisfies

$$|R_n(x)| \leq \frac{M}{n!}|x - a|^n \quad \text{(for } |x - a| < r\text{)}.$$

55.

$$\int_0^{\sqrt{10}} \frac{x^3}{\sqrt{10 + x^2}} \, dx = \int_0^{\pi/4} \frac{(10^{3/2} \tan^3 \theta)(\sqrt{10} \sec^2 \theta)}{\sqrt{10 + 10 \tan^2 \theta}} \, d\theta = 10^{3/2} \int_0^{\pi/4} \tan^3 \theta \sec \theta \, d\theta.$$

56. $(n!)^2 = (2n)!$

57. $\sum_{n=0}^{\infty} (-1)^n x^{5n+2}$ is a power series.

58. The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

59. $\arccos(x) = \cos^{-1}(x) = \frac{1}{\cos(x)}$

60. $\frac{1}{x^2(x^2+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+2}$ is a valid partial fraction decomposition.

61. If $\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence $R = 7$, then the series $\sum_{n=0}^{\infty} (n + 1)a_{n+1}x^n$ has radius of convergence $R = 7$.

62. The harmonic series is conditionally convergent.

63. $y'' + 3x^2 y' - 4 \ln(x) y = x^2$ is a linear differential equation.

64.

$$(1 + x)^{-1/3} = \frac{1}{(1 + x)^{1/3}} = \frac{1}{1 + x^{1/3}} = \sum_{n=0}^{\infty} (-x^{1/3})^n.$$

65. Since $\frac{\cos^2 n}{n^2} \leq \frac{1}{n^2}$ for all $n \geq 1$, the series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$ converges.

66. $\int \sin(1 - x) \, dx = -\cos(1 - x) + C$.

67. $\sum_1^{\infty} \frac{\cos n}{\sqrt{n}}$ converges by the alternating series test.

68.

$$\int \frac{1}{x^2 + 2x + 4} \, dx = \int \frac{1}{(x + 1)^2 + 3} \, dx = \int \frac{1}{u^2 + 3} \, du \quad \text{where } u = x + 1.$$

69. $1 + 2 + 3 + 4 + \cdots + 99 + 100 + \sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.

70. $y' = xy + x$ is a separable differential equation.

71. A boundary-value problem always has a solution.

72. $\sum_{n=2}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n-1} \right) = -\frac{3}{2}$.

73. If $\sum a_n$ is convergent, then $\sum (-1)^n a_n$ is convergent.

74. For the differential equation $y' + 2xy = 3x$, one possible integrating factor is $I(x) = e^{x^2}$.

75.

$$\cos\left(\arctan\left(\frac{x}{2}\right)\right) = \frac{\sqrt{x^2 + 4}}{2}.$$

76. The series $\sum_{n=1}^{\infty} n^2 e^{-n}$ converges by the Integral Test.
77. The integral $\int x^2 e^{x^2}\,dx$ is impossible to evaluate exactly.
78. The arc length of $y = 10x^3$ from $x = 1$ to $x = 2$ is given by $\int_1^2 \sqrt{1 + 30x^2}\,dx$.
79. If the radius of convergence of $\sum_{n=0}^{\infty} a_n x^n$ is $R = 2$, then the interval of convergence is $(-2, 2)$.
80. If you approximate $\int_0^1 e^{x^2}\,dx$ with the Trapezoidal Rule with 2 subdivisions, you get $T_2 = \frac{1}{4}\left(1 + 2e^{1/4} + e\right) \approx 1.57$. Therefore, $\int_0^1 e^{x^2}\,dx \geq 1.4$.
81. The initial-value problem $y' = \sqrt{y},\ y(0) = 0$ has only one solution: $y = 0$.
82. $\sum_{n=1}^{\infty} (-1)^{n+1} e^{-n} \leq \frac{1}{e}$.