1. $\sum_{n=4}^{\infty}\left(\frac{1}{3}\right)^n$ converges.
   Solution: **True.** This is a geometric series with $r = \frac{1}{3} < 1$.

2. If $\sum a_n$ is absolutely convergent, then $\sum a_n$ is convergent.
   Solution: **True.** This is a known property of absolute convergence.

3. $\int e^{2x}\,dx = \frac{C}{2}e^{2x}$.
   Solution: **False.** $\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C$.

4. If the nonhomogeneous part of a second-order, linear, constant-coefficient differential equation is $g(x) = e^x$, then $y_p(x) = Ae^x$ is always a valid form of the particular solution.
   Solution: **False.** If the homogeneous solution contains and e^x term, then you must boost the particular solution by x or x^2.

5. If $f(x) \le g(x)$ and $\int_0^{\infty} g(x)\,dx$ diverges, then $\int_0^{\infty} f(x)\,dx$ diverges.
   Solution: **False.** If the bigger function diverges, this says nothing about the smaller function. Take $f(x) = 0$ and $g(x) = 1$.

6. $\int \sin(3x)\cos(2x)\,dx = \frac{1}{2}\int [\sin(5x) + \sin(x)]\,dx$.
   Solution: **True.** This is a product-to-sum identity.

7. $\sum_{n=0}^{\infty}\frac{(-1)^n}{n!} = \frac{1}{e}$.
   Solution: **True.** This is the Taylor series expansion of e^x with $x = -1$.

8. On the first day of the semester, Oski starts an ant farm. Let $A(t)$ be the number of ants in Oski's ant farm t days after the first day of the semester. If we model the ant population using the natural growth model with growth rate constant $k = 3$, then Oski's ant farm population adds 3 ants per day.
   Solution: **False.** Oski's ant farm population grows exponentially.

9. Since $-\frac{1}{n} \le \frac{\sin(n)}{n} \le \frac{1}{n}$ and $\lim_{n\to\infty} -\frac{1}{n} = \lim_{n\to\infty}\frac{1}{n} = 0$, it follows that $\lim_{n\to\infty}\frac{\sin(n)}{n} = 0$.
   Solution: **True.** This is the squeeze theorem for sequences.

10. $\int_{-\infty}^{\infty} e^{-|x|}\,dx$ converges.
    Solution: **True.** $\int_{-\infty}^{\infty} e^{-|x|}\,dx = \int_{-\infty}^{0} e^x\,dx + \int_0^{\infty} e^{-x}\,dx = 2$.

11. $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$ converges by the Ratio Test.
    Solution: **False.** The Ratio Test is inconclusive. The alternating harmonic series converges by the alternating series test.

12. Euler's Method gives an exact solution to a differential equation.
    Solution: **False.** Euler's Method is only an approximation.

13. $\int_1^\infty \frac{1}{x}\,dx$ converges.

Solution: **False.** $\int_1^\infty \frac{1}{x}\,dx = \ln|x|\Big|_1^\infty = \infty$.

14. $\sum_{n=1}^\infty \frac{1}{n^{3/2}-2}$ converges by the Direct Comparison Test to $\sum_{n=1}^\infty \frac{1}{n^{3/2}}$.

Solution: **False.** Direct comparison doesn't work here because $\frac{1}{n^{3/2}-2} > \frac{1}{n^{3/2}}$. Instead, you can use the Limit Comparison Test.

15. The logistic growth equation is a linear differential equation.

Solution: **False.** The logistic equation $\frac{dP}{dt} = kP(1-P/M)$ is separable but not linear.

16. $\int_{-\pi/2}^{\pi/2} \csc(|x|)\,dx = \ln|\csc(\pi/2)-\cot(\pi/2)| - \ln|\csc(\pi/2)-\cot(\pi/2)| = 0$.

Solution: **False.** $\csc(x)$ diverges at $x=0$, so you must break it into two integrals. The improper integral diverges.

17. If we use $u = x^3$, then $\int_0^2 x^2\cos(x^3)\,dx = \int_0^2 \frac{1}{3}\cos(u)\,du$.

Solution: **False.** The bounds of the integral also need to change.

18. If the interval of convergence of a power series is $[0,4)$, then the radius of convergence is $R = 2$.

Solution: **True.** The radius of convergence is half the length of the interval.

19. $y' = y - 2$ is a separable differential equation.

Solution: **True.** It is $y' = g(x)f(y)$, where $g(x) = 1$ and $f(y) = y-2$.

20.

$$\int_0^{\pi/2} \sin^5(x)\cos^2(x)\,dx = \int_0^1 (1-u^2)^2u^2\,du.$$

Solution: **True.** This uses the substitution $u = \cos(x)$ together with the trig identity $\sin^2(x) = 1-\cos^2(x)$.

21. You can use integration by parts on improper integrals.

Solution: **True.** All usual integration techniques can be used on improper integrals.

22. $\sum_{n=1}^\infty (-1)^n n^n$ diverges by the Root Test.

Solution: **True.** Applying the root test gives n, which diverges to ∞.

23. $\sum_{n=1}^\infty \left(\frac{1}{2}\right)^n = 2$.

Solution: **False.** The geometric series formula starts at 0. $\sum_{n=1}^\infty \left(\frac{1}{2}\right)^n = 2-1 = 1$.

24. The third-order Taylor polynomial of $\sin x$ centered at $x=0$ is $T_3(x) = x - \frac{x^3}{6}$.

Solution: **True.** The third-order Taylor polynomial goes up to the x^3 term.

25. Since $\lim_{n\to\infty} \frac{3\sqrt{n}+2+n^{-1}}{\sqrt{n}} = 3$, the series $\sum_{n=1}^{\infty} \frac{1}{3\sqrt{n}+2+n^{-1}}$ converges because $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges.

    Solution: **False.** The series diverges because $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges; it is a p-series with $p = 1/2 < 1$.

26. $\frac{x^2}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$ is a valid partial fraction decomposition.

    Solution: **False.** The degree of the numerator and denominator are both 2, so you must divide first.

27. The center of mass of the region between $y = x+1$, $y = 0$, $x = -1$, and $x = 1$ is in the first quadrant.

    Solution: **True.** You can compute this using the formula or draw the graph to see that more of the region is in the first quadrant than the second quadrant.

28. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\cos(\pi n)n^2}$ converges by the alternating series test.

    Solution: **False.** Since $\cos(\pi n) = (-1)^n$, this series is actually equivalent to $\sum_{n=1}^{\infty} \frac{1}{n^2}$, which is not an alternating series. This is a convergent p-series.

29. In the method of variation of parameters, if the homogeneous solution is $y_h(x) = C_1 e^{-x}\cos(3x) + C_2 e^{-x}\sin(3x)$, then the particular solution is of the form $y_p(x) = u_1(x)e^{-x}\cos(3x) + u_2(x)e^{-x}\sin(3x)$.

    Solution: **True.** $y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$.

30. A spring with a 3-kg mass and a damping constant 10 kg/s can be held stretched 0.5 meters beyond its natural length by a force of 2 newtons. The spring is then stretched 4 meters beyond its natural length and released with zero velocity. The spring constant is $k = 4/2 = 2$ newtons per meter.

    Solution: **False.** The spring constant is $k = 2/0.5 = 4$ newtons per meter.

31. $dG/dt = 3G(1-0.01G)$ describes logistic growth with carrying capacity 100.

    Solution: **True.** $dG/dt = 3G(1-0.01G) = 3G(1-G/100)$.

32. $\int \frac{1}{x^2}\,dx = \ln(x^2) + C$.

    Solution: **False.** $\int \frac{1}{x^2}\,dx = \frac{1}{x} + C$.

33. If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ is divergent.

    Solution: **False.** Let $a_n = 1$ and $b_n = -1$. This does hold for convergent sequences: if $\{a_n\}$ and $\{b_n\}$ are convergent, then $\{a_n + b_n\}$ is convergent.

34. The Lotka-Volterra predator-prey equations always have exactly one equilibrium solution.

    Solution: **False.** There are usually two equilibrium solutions, one of which is the solution $(0,0)$.

35. If the ratio test on $\sum_{n=0}^{\infty} a_n x^n$ says that the power series converges for $\left|x - \frac{1}{3}\right| < 1$, then the radius of convergence is $R = \frac{1}{3}$.

    Solution: **False.** The radius of convergence is 1. The interval of convergence is centered at $x = 1/3$.

36. Every bounded, monotonic sequence is convergent.

    Solution: **True.** This is the Monotone Sequence Theorem.

37. $\sum_{n=0}^{\infty} \cos(n^2)\frac{3^n}{4^n}$ is absolutely convergent.

    Solution: **True.** Take the absolute value and compare to the geometric series with $r = 3/4$.

38. If $y^{1/2} = \sin^2(x) + C$, then $y = \sin^4(x) + C^2$.

    Solution: **False.** $y = \left(\sin^2(x) + C\right)^2$.

39. A tank contains 1000 L of pure water. Brine that contains 0.05 kg of salt per liter of water enters the tank at a rate of 5 L/min. Brine that contains 0.04 kg of salt per liter of water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at a rate of 15 L/min. If $S(t)$ is the kilograms of salt in the tank after t minutes, the initial-value problem that describes this situation is

$$\frac{dS}{dt} = 0.25 - 0.015S, \quad S(0) = 0.$$

    Solution: **False.** There is an additional 0.4 kg/min, so the rate in is 0.65 kg/min.

40. $\sum_{n=0}^{\infty}(-3)^n = \frac{1}{1-(-3)} = \frac{1}{4}$.

    Solution: **False.** This is a geometric series with $r = -3$, which diverges. The formula only applies for $|r| < 1$.

41. The Maclaurin series of $f(x)$ is the Taylor series of $f(x)$ centered at $x = 0$.

    Solution: **True.** This is the definition of a Maclaurin series.

42. $\frac{1}{3-x} = \frac{1}{1-(x/3)} = \sum_{n=0}^{\infty}(x/3)^n$.

    Solution: **False.** In fact, $\frac{1}{3-x} = \frac{1/3}{1-(x/3)} = \frac{1}{3}\sum_{n=0}^{\infty}(x/3)^n$.

43. $\sum_{n=1}^{\infty} n^{4/3}$ converges since $p = 4/3 > 1$.

    Solution: **False.** This is $p = -4/3 < 1$, so the series diverges.

44. $2 \cdot 4 \cdot 6 \cdots (2m) = 2^m m!$

    Solution: **True.** There are m factors, and you can factor out a 2 from each of them.

45. Since $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$, the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges.

    Solution: **False.** If the limit of the sequence is 0, you cannot say anything. In fact, this is a divergent p-series with $p = 1/2$.

46. The auxiliary equation for $y'' + 2y' = 0$ is $r^2 + 2 = 0$.

    Solution: **False.** The auxiliary equation is $r^2 + 2r = 0$.

47. If the Ratio Test is inconclusive, then you can still use a different test to determine if the series converges.

    Solution: **True.** If the Ratio Test is inconclusive, you can still try other tests (e.g. comparison, alternating, etc.)

48. $\sum_{n=1}^{\infty}(-1)^n n$ diverges by the Root Test.

Solution: **False.** The Root Test is inconclusive for this test. The series still diverges by the Divergence Test.

49. The radius of convergence of a power series can be zero.

Solution: **True.** Consider the power series $\sum_{n=0}^{\infty} n!x^n$.

50. If the recursion relation of a series solution $\sum_{n=0}^{\infty} c_n x^n$ to a second-order linear differential equation is $c_{n+2} = \frac{1}{2}c_n$, then the general solution is

$$y(x) = c_0 \sum_{m=0}^{\infty} \frac{1}{2^m} x^{2m} + c_1 \sum_{m=0}^{\infty} \frac{1}{2^m} x^{2m+1}.$$

Solution: **True.** The even and odd terms are separate, and writing out the first several terms of the recursion relation yields the above series.

51. If the homogeneous solution of a nonhomogeneous, second-order, linear, constant-coefficient differential equation is $y_h(x) = C_1 e^{2x} + C_2 e^{-x}$ and the nonhomogeneous part is $g(x) = x\cos x$, then a valid guess for a particular solution is $y_p(x) = (Ax + B)(C\cos x + D\sin x)$.

Solution: **False.** $y_p(x) = (Ax + B)\cos x + (Cx + D)\sin x$.

52. $\int \tan x \, dx = \sec^2 x + C$.

Solution: **False.** $\int \tan x \, dx = \ln|\sec x| + C$.

53. $y(x) = e^x - e^{-x+2}$ is a solution to $y'' - y = 0$.

Solution: **True.** Here, $C_1 = 1$ and $C_2 = -e^2$.

54. Taylor's Inequality states that if $|f^{(n+1)}(x)| \leq M$ for $|x - a| < r$, then the remainder $R_n(x) = f(x) - T_n(x)$ of the Taylor series satisfies

$$|R_n(x)| \leq \frac{M}{n!}|x - a|^n \quad (\text{for } |x - a| < r).$$

Solution: **False.** $|R_n(x)| \leq \frac{M}{(n+1)!}|x - a|^{n+1}$.

55.

$$\int_0^{\sqrt{10}} \frac{x^3}{\sqrt{10 + x^2}}\, dx = \int_0^{\pi/4} \frac{(10^{3/2}\tan^3\theta)(\sqrt{10}\sec^2\theta)}{\sqrt{10 + 10\tan^2\theta}}\, d\theta = 10^{3/2}\int_0^{\pi/4} \tan^3\theta \sec\theta \, d\theta.$$

Solution: **True.** This is trig substitution with $x = \sqrt{10}\tan\theta$.

56. $(n!)^2 = (2n)!$

Solution: **False.** $(n!)^2$ can't be simplified further.

57. $\sum_{n=0}^{\infty}(-1)^n x^{5n+2}$ is a power series.

Solution: **True.** This is still $\sum_{n=0}^{\infty} c_n x^n$, but most of the c_n's are zero.

58. The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

    Solution: **True.** Use integral test.

59. $\arccos(x) = \cos^{-1}(x) = \frac{1}{\cos(x)}$

    Solution: **False.** $\arccos(x) = \cos^{-1}(x) \neq \frac{1}{\cos(x)} = \sec(x)$.

60. $\frac{1}{x^2(x^2+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+2}$ is a valid partial fraction decomposition.

    Solution: **True.** This satisfies the rules for repeated linear factors and irreducible quadratics.

61. If $\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence $R = 7$, then the series $\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n$ has radius of convergence $R = 7$.

    Solution: **True.** The derivative of $\sum_{n=0}^{\infty} a_n x^n$ is $\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n$, and the radius of convergence of the derivative is the same.

62. The harmonic series is conditionally convergent.

    Solution: **False.** The *alternating harmonic series* is conditionally convergent.

63. $y'' + 3x^2y' - 4\ln(x)y = x^2$ is a linear differential equation.

    Solution: **True.** This is of the form $p(x)y'' + q(x)y' + r(x)y = g(x)$.

64. $$(1+x)^{-1/3} = \frac{1}{(1+x)^{1/3}} = \frac{1}{1+x^{1/3}} = \sum_{n=0}^{\infty}(-x^{1/3})^n.$$

    Solution: **False.** This is in fact a binomial series with $k = -1/3$.

65. Since $\frac{\cos^2 n}{n^2} \leq \frac{1}{n^2}$ for all $n \geq 1$, the series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$ converges.

    Solution: **True.** These series are both positive, and the direct comparison test holds.

66. $\int \sin(1-x)\,dx = -\cos(1-x) + C$.

    Solution: **False.** The substitution $u = 1-x$ means $du = -dx$, so the solution is $\cos(1-x)+C$ (positive sign).

67. $\sum_{1}^{\infty} \frac{\cos n}{\sqrt{n}}$ converges by the alternating series test.

    Solution: **False.** $\cos n$ is not strictly alternating. The alternating series test only works for terms like $(-1)^n, (-1)^{n+1}$, etc.

68. $$\int \frac{1}{x^2+2x+4}\,dx = \int \frac{1}{(x+1)^2+3}\,dx = \int \frac{1}{u^2+3}\,du \quad \text{where } u = x+1.$$

    Solution: **True.** This is the complete-the-square formula.

69. $1 + 2 + 3 + 4 + \cdots + 99 + 100 + \sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.

    Solution: **True.** $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent, and adding a finite number of terms makes no difference.

70. $y' = xy + x$ is a separable differential equation.

    Solution: **True.** This can be re-written as $y' = x(y + 1)$.

71. A boundary-value problem always has a solution.

    Solution: **False.** A boundary-value problem can have no solution.

72. $\sum_{n=2}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n-1} \right) = -\frac{3}{2}$.

    Solution: **True.** This is a telescoping series and the only terms that don't cancel are -1 and $-1/2$.

73. If $\sum a_n$ is convergent, then $\sum (-1)^n a_n$ is convergent.

    Solution: **False.** This doesn't work with $a_n = (-1)^n \frac{1}{n}$. This statement is true if a_n is always positive.

74. For the differential equation $y' + 2xy = 3x$, one possible integrating factor is $I(x) = e^{x^2}$.

    Solution: **True.** The integrating factor is $I(x) = e^{\int 2x\, dx}$.

75. $$\cos\left(\arctan\left(\frac{x}{2}\right)\right) = \frac{\sqrt{x^2 + 4}}{2}.$$

    Solution: **False.** The numerator and denominator are flipped: should be $\frac{2}{\sqrt{x^2+4}}$.

76. The series $\sum_{n=1}^{\infty} n^2 e^{-n}$ converges by the Integral Test.

    Solution: **True.** This satisfies all three conditions for the Integral Test (continuous, positive, eventually decreasing) and the integral converges.

77. The integral $\int x^2 e^{x^2}\, dx$ is impossible to evaluate exactly.

    Solution: **False.** $u = x^2$ is a valid substitution. On the other hand, the integral $\int e^{x^2}\, dx$ is impossible to evaluate exactly.

78. The arc length of $y = 10x^3$ from $x = 1$ to $x = 2$ is given by $\int_1^2 \sqrt{1 + 30x^2}\, dx$.

    Solution: **False.** Don't forget to square the derivative. The arc length is $\int_1^2 \sqrt{1 + 900x^4}\, dx$.

79. If the radius of convergence of $\sum_{n=0}^{\infty} a_n x^n$ is $R = 2$, then the interval of convergence is $(-2, 2)$.

    Solution: **False.** The endpoints could be included in the interval.

80. If you approximate $\int_0^1 e^{x^2}\,dx$ with the Trapezoidal Rule with 2 subdivisions, you get $T_2 = \frac{1}{4}\left(1 + 2e^{1/4} + e\right) \approx 1.57$. Therefore, $\int_0^1 e^{x^2}\,dx \geq 1.4$.

    Solution: **True.** Applying the Trapezoidal Rule error bound, the remainder is bounded by $e/16 \approx 0.17$, so the true value of the integral cannot be smaller than $T_2 - 0.17 \approx 1.57 - 0.17 = 1.4$. The true value is actually around 1.46.

81. The initial-value problem $y' = \sqrt{y},\ y(0) = 0$ has only one solution: $y = 0$.

    Solution: **False.** $y = \frac{x^2}{4}$ is also a solution.

82. $\sum_{n=1}^{\infty} (-1)^{n+1} e^{-n} \leq \frac{1}{e}$.

    Solution: **True.** By the Alternating Series Estimation Theorem with $n = 1$, the sum cannot be larger than the absolute value of the first term.