1. $\sum_{n=4}^{\infty} \left(\frac{1}{3}\right)^n$ converges.

2. The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

3. The Maclaurin series of $f(x)$ is the Taylor series of $f(x)$ centered at $x = 0$.

4. If $\sum a_n$ is absolutely convergent, then $\sum a_n$ is convergent.

5. $\int e^{2x}\, dx = \frac{C}{2} e^{2x}$.

6. $y' = y - 2$ is a separable and autonomous differential equation.

7. If the interval of convergence of a power series is $[0, 4)$, then the radius of convergence is $R = 2$.

8. $$\int_0^{\pi/2} \sin^5(x) \cos^2(x)\, dx = \int_0^1 (1 - u^2)^2 u^2\, du.$$

9. If $f(x) = \sum_{n=0}^{\infty} 2^n (x-1)^n$, then $f^{(2024)}(1) = 2024! \cdot 2^{2024}$.

10. $\sum_{n=1}^{\infty} (-1)^n n^n$ diverges.

11. $\frac{x^2}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$ is a valid partial fraction decomposition.

12. If the nonhomogeneous part of a second-order, linear, constant-coefficient differential equation is $g(x) = e^x$, then $y_p(x) = Ae^x$ is always a valid form of the particular solution.

13. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\cos(\pi n) n^2}$ converges by the alternating series test.

14. The auxiliary equation for $y'' + 2y' = 0$ is $r^2 + 2 = 0$.

15. A boundary-value problem always has exactly one solution.

16. $\int \frac{1}{x^2}\, dx = \ln(x^2) + C$.

17. If the ratio test on $\sum_{n=0}^{\infty} a_n x^n$ says that the power series converges for $\left|x - \frac{1}{3}\right| < 1$, then the radius of convergence is $R = \frac{1}{3}$.

18. $\sum_{n=0}^{\infty} \cos(n^2) \frac{3^n}{4^n}$ is absolutely convergent.

19. If $y^{1/2} = \sin^2(x) + C$, then $y = \sin^4(x) + C^2$.

20. If $\sum_{n=0}^{\infty} c_n(-3)^n$ converges, then it is possible that $\sum_{n=0}^{\infty} c_n$ diverges.

21. $\frac{1}{3-x} = \frac{1}{1-(x/3)} = \sum_{n=0}^{\infty}(x/3)^n$.

22. The Taylor series of $f(x) = e^x$ centered at $x = 2$ is $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n!}$.

23. $\sum_{n=1}^{\infty} n^{4/3}$ converges since $p = 4/3 > 1$.

24. Since $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$, the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges.

25. The slope field below (Figure 1) describes $y' = 2 - y$.

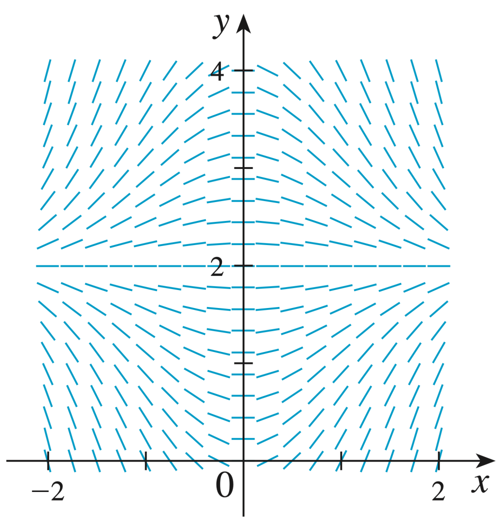

Figure 1: (Credit: Stewart Calculus)

26. $3x^2y' - 4\ln(x)y = x^2$ is a first-order linear differential equation.

27. $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$ converges.

28. $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}-2}$ converges.

29. If the Ratio Test is inconclusive, then you can still use a different test to determine if the series converges.

30. $\sum_{n=1}^{\infty}(-1)^n n$ diverges by the Root Test.

31. The radius of convergence of a power series can be zero.

32. Consider the differential equation $y' = y^2 - 1$. If $y(0) = 0$, then $\lim_{x\to\infty} y(x) = \infty$.

33. If the complementary solution of a nonhomogeneous, second-order, linear, constant-coefficient differential equation is $y_c(x) = C_1 e^{2x} + C_2 e^{-x}$ and the nonhomogeneous part is $g(x) = x\cos x$, then a valid guess for a particular solution is $y_p(x) = (Ax + B)(C\cos x + D\sin x)$.

34. $\int \tan x\, dx = \sec^2 x + C$.

35. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$.

36.

$$\int_0^{\sqrt{10}} \frac{x^3}{\sqrt{10+x^2}}\, dx = \int_0^{\pi/4} \frac{(10^{3/2}\tan^3\theta)(\sqrt{10}\sec^2\theta)}{\sqrt{10+10\tan^2\theta}}\, d\theta = 10^{3/2}\int_0^{\pi/4} \tan^3\theta \sec\theta\, d\theta.$$

37. $(n!)^2 = (2n)!$

38. $\sum_{n=0}^{\infty}(-1)^n x^{5n+2}$ is a power series.

39. Since $\lim_{n\to\infty} \frac{3\sqrt{n}+2+n^{-1}}{\sqrt{n}} = 3$, the series $\sum_{n=1}^{\infty} \frac{1}{3\sqrt{n}+2+n^{-1}}$ converges because $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges.

40. If $\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence $R = 7$, then the series $\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n$ has radius of convergence $R = 7$.

41. $\int_1^{\infty} \frac{1}{x}\, dx$ converges.

42. $\arccos(x) = \cos^{-1}(x) = \frac{1}{\cos(x)}$

43. $\frac{1}{x^2(x^2+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+2}$ is a valid partial fraction decomposition.

44. The harmonic series is conditionally convergent.

45.

$$(1+x)^{-1/3} = \frac{1}{(1+x)^{1/3}} = \frac{1}{1+x^{1/3}} = \sum_{n=0}^{\infty}(-x^{1/3})^n.$$

46. Since $\frac{\cos^2 n}{n^2} \leq \frac{1}{n^2}$ for all $n \geq 1$, the series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$ converges.

47. If the recursion relation of a series solution $\sum_{n=0}^{\infty} c_n x^n$ to a second-order linear differential equation is $c_{n+2} = \frac{1}{2} c_n$, then the general solution is

$$y(x) = c_0 \sum_{k=0}^{\infty} \frac{1}{2^k} x^{2k} + c_1 \sum_{k=0}^{\infty} \frac{1}{2^k} x^{2k+1}.$$

48. $\int \sin(1-x)\,dx = -\cos(1-x) + C$.

49. $\sum_1^{\infty} \frac{\cos n}{\sqrt{n}}$ converges by the alternating series test.

50.
$$\int \frac{1}{x^2+2x+4}\,dx = \int \frac{1}{(x+1)^2+3}\,dx = \int \frac{1}{u^2+3}\,du \text{ where } u = x+1.$$

51. $y' = xy + x$ is a separable differential equation.

52. $\sum_{n=2}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n-1}\right) = -\frac{3}{2}$.

53. If $\sum a_n$ is convergent, then $\sum (-1)^n a_n$ is convergent.

54. For the differential equation $y' + 2xy = 3x$, one possible integrating factor is $A(x) = e^{x^2}$.

55.
$$\cos\left(\arctan\left(\frac{x}{2}\right)\right) = \frac{\sqrt{x^2+4}}{2}.$$

56. The series $\sum_{n=1}^{\infty} n^2 e^{-n}$ converges.

57. If the radius of convergence of $\sum_{n=0}^{\infty} a_n x^n$ is $R = 2$, then the interval of convergence is $(-2, 2)$.

58. All solutions to the initial-value problem $y' = \sqrt{y}$ are of the form $y = \left(\frac{1}{2}x + C\right)^2$.

59. If $\sum_{n=0}^{\infty} (-1)^n c_n 2^n$ is convergent, then $\sum_{n=0}^{\infty} \frac{c_n}{4^n}$ is convergent.

60. If $f(x) = \sum_{n=0}^{\infty} \frac{3^n}{n!} x^{n+2}$, then $f^{(500)}(0) = 3^{500}$.